Indian Statistical Institute, Bangalore
B. Math (II) Second Semester 2018-19
Semester Examination : Statistics (II)

Date: 08-05-2019 **Maximum Score 60** **Duration: 3 Hours**

1. An entrepreneur who deals in washing machines is interested in finding a suitable guarantee period for her product. She is willing to set the guarantee period to 7 years provided $\tau(\theta)$, the probability that a washing machine fails before completing 7 years, is not too large. Suppose the lifetime X of her product is known to have *exponential distribution* with *pdf* $f_X(x|\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}} I_{(0,\,\infty)}(x)$, $\theta > 0$ being unknown. Suppose n such randomly chosen machines are put to survival test and that $X_1, X_2, \cdots, X_n$ are their lifetimes. Can you help the entrepreneur to find $\tau(\theta)$ and estimate it based on $X_1, X_2, \cdots, X_n$. Is your estimator *uniformly minimum variance unbiased estimator (UMVUE)* for estimating $\tau(\theta)$? If yes, substantiate. If no, obtain *UMVUE* for $\tau(\theta)$.

   $[3 + 10 = 13]$

2. Suppose that the number of minutes Tarannum must wait for her bus each morning has uniform distribution on the interval $[0, \theta]$, where the value of the endpoint $\theta > 0$ is unknown. Suppose also that the *prior pdf* for θ is $\pi(\theta) = \frac{192}{\theta^4} I_{[4,\infty)}(\theta)$. If $X_1, X_2, ..., X_n$ are the observed waiting times on n randomly chosen mornings, then find the *Bayes estimator* under squared error loss. If the observed waiting times on three randomly chosen mornings were $5, 3,$ and 8 minutes, find the *posterior pdf*, and *posterior mean.*

   $[8 + 2 + 3 = 13]$

3. Let $X_1, X_2, \cdots, X_n$ be a random sample from $Poisson(\lambda)$, $\lambda > 0$. Derive likelihood ratio test at $\alpha = 0.05$ for testing the hypothesis

   $$H_0 : \lambda \geq \lambda_0 \ versus \ H_1 : \lambda < \lambda_0.$$

   Find p value. Obtain 90% *lower confidence bound* (LCB) for λ.

   $[8 + 2 + 3 = 13]$

4. To study the effect of cigarette smoking on platelet aggregation researchers drew blood samples from 11 individuals before and after they smoked a cigarette and measured the percentage of blood platelet aggregation as given in the table below. Platelets are involved in the formation of blood clots, and it is known that the smokers suffer more often from disorders involving blood clots like arterial thrombosis than do nonsmokers. Do the following data support, at $\alpha = 0.05$, the hypothesis that smoking increases platelet aggregation? State clearly the assumptions you make. Find p-value. Find 90% *upper confidence bound* (UCB) for the mean increase in platelet aggregation.

Sr. No. $\longrightarrow$ % $\downarrow$	1	2	3	4	5	6	7	8	9	10	11
Before	25	25	27	44	30	67	53	53	52	60	28
After	27	29	37	56	46	82	57	80	61	59	43

$[8 + 2 + 3 = 13]$

[*PTO*]

5. In the Cricket World Cup the following procedure may be used to decide a *TOSS*. The caller has two options Heads and Tails. If the caller calls Heads(Tails) a coin is independently tossed maximum of 5 times until either 3 Heads or 3 Tails are observed. If 3 Heads(3 Tails) are observed then the caller wins the *TOSS* otherwise s/he loses. If θ is the probability of heads in a usual single toss then find the probability of caller calling Heads and winning the *TOSS*.

   The following experiment was conducted with the special gold coin to be used for the *TOSS*. The coin was tossed independently 5 times and X : the number of heads was recorded. This was repeated 64 times and the following data was obtained.

No of Heads	*Frequency*
0	01
1	08
2	23
3	16
4	13
5	03

   Set up and carry out a test at $\alpha = 0.05$, to validate the *fairness* of the *TOSS* based on the collected data. Also report the *p value.*

   $[3 + 10 = 13]$